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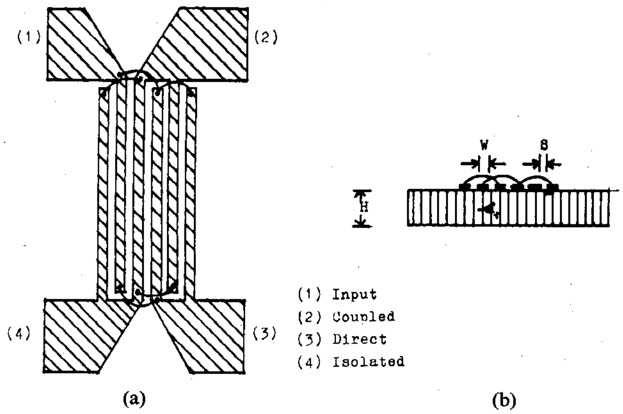

Fig. 1. (a) Conductor pattern and (b) cross section of a six-strip Lange coupler.

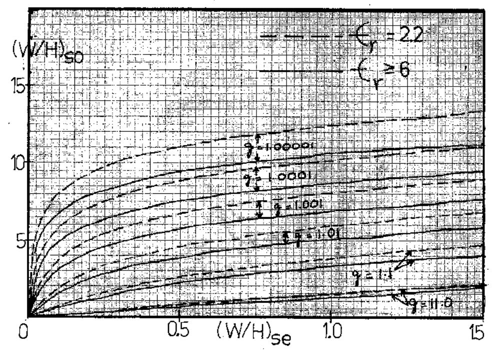

Fig. 2. g as a function of $(W/H)_{so}$ and $(W/H)_{se}$ (4).

Synthesis of Lange Couplers

R. M. OSMANI

***Abstract*—This paper shows that it is possible to synthesize Lange couplers directly and thereby save considerable computing time. The procedure outlined is essentially based on two available techniques: Ou's analysis of the Lange coupler and Akhtarzad's design method for a pair of coupled microstrip lines. Including a correction in the latter for single strip shape ratios less than unity, is significant. The described approach compares favorably with existing iterative methods and was used to obtain reasonably good performances on a low dielectric constant laminate.**

I. Introduction

Tight coupling in microstrip became feasible with the introduction of the Lange coupler [1] in 1969. Recent papers on Lange couplers [2]–[7] are based on an analysis of the coupler and therefore a number of iterations are needed to establish a new design. Analysis is more suitable for confirming or optimizing an initial design [8]; synthesis is preferable for determining the dimensions of a coupler for any given requirement of coupling value, dielectric constant, terminating impedance, and number of strips.

In this paper, a direct synthesis procedure for Lange couplers is outlined. Ou's method is first used to determine odd and even-mode impedances of any adjacent pair of lines in the array [9]. Final dimensions are obtained by applying the synthesis technique of Akhtarzad *et al.* [10]. All equations given here are simple enough to be solved without a computer. Good agreement with more involved analytical methods is shown by both theoretical and experimental results. An empirical correction for finite conductor thickness given by Presser [5] did not explain the extent of overcoupling observed for couplers fabricated on a low dielectric constant substrate.

II. Synthesis

The conductor pattern and cross section of a six-strip Lange coupler is shown in Fig. 1 to illustrate the relevant dimensions. All strips are assumed to be equispaced, to be of equal width, and also to be quarter-wavelength long at the center frequency.

For any adjacent pair of lines in the array, the odd- and even-mode impedances Z_{0o} and Z_{0e}, respectively, are obtained by solving the analysis equations of Ou [9]. Although solutions are available [5], [11] it is shown in Appendix that simpler expressions for Z_{0o} and Z_{0e} are possible:

$$Z_{0o} = Z_0\left(\frac{1-c}{1+c}\right)^{1/2} \cdot \frac{(k-1)\cdot(1+q)}{(c+q)+(k-1)\cdot(1-c)} \tag{1}$$

$$Z_{0e} = Z_{0o}\frac{(c+q)}{(k-1)\cdot(1-c)} \tag{2}$$

where

$$q = \left[c^2 + (1-c^2)\cdot(k-1)^2\right]^{1/2}$$

k is the even number of strips, Z_0 is the terminating impedance and c is the voltage coupling coefficient.

The synthesis technique of Akhtarzad *et al.* [10] for a pair of coupled microstriplines can now be applied. In order to relate Z_{0o} and Z_{0e} to the physical dimensions of the coupler, single strip shape ratios $(W/H)_{so}$ and $(W/H)_{se}$ corresponding to the impedances $Z_{0o}/2$ and $Z_{0e}/2$, respectively, are first calculated. In the original method this was done by using Wheeler's equation for wide strips [10]. For shape ratios less than unity, this causes an appreciable error [12]. The procedure adopted here avoids this error by using Wheeler's equation valid for both wide and narrow strips [13]. The required shape ratios $(W/H)_{so}$ and $(W/H)_{se}$ are determined by substituting the known values of

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TABLE I
COMPARISON OF DATA ($Z_0 = 50\ \Omega$; $T = 0$)

				Reported dimensions		Calculated dimensions		Calculated performance for reported dimensions	
Reference	ϵ_r	k	Coupling in decibles	W/H	S/H	W/H	S/H	Coupling in decibles	Z_0 in ohms
[3]	3.78	4	5.0	0.425	0.20	0.41	0.19	5.3	49
[7]	3.8	6	1.5	0.10	0.03	0.07	0.028	1.5	46
[1]	9.6	4	3.0	0.107	0.071	0.08	0.073	2.9	46
[16]	9.6	4	6.5	0.15	0.25	0.12	0.32	5.4	47
[5]	10.0	4	3.0	0.075	0.075	0.075	0.073	3.0	50
[14]	16.0	2	10.0	0.5	0.3	0.48	0.31	10.1	49

Z_{0o} and Z_{0e}, respectively, in equation (3) by turn

$$(W/H)_{so,se} = \frac{8}{p}\cdot\left[\frac{p\cdot(7+4/\epsilon_r)}{11.0} + \frac{1+1/\epsilon_r}{0.81}\right]^{1/2} \tag{3}$$

where

$$p = \left[\exp\left\{\frac{Z_{0o,0e}}{84.8}(\epsilon_r+1)^{1/2}\right\} - 1\right]$$

where ϵ_r is the relative dielectric constant. The single strip shape ratios thus calculated are related to the normalized width (W/H) and spacing (S/H) of the Lange coupler [10]. The two synthesis equations of reference [10] can be combined and expressed as

$$(W/H)_{so} = \frac{2}{\pi}\cosh^{-1}\left[\frac{(g+1)\cdot f - 2}{(g-1)}\right] + r\cdot\cosh^{-1}\left[\frac{\cosh^{-1}\left\{\frac{(g+1)\cdot f}{2} + \frac{(g-1)}{2}\right\}}{\cosh^{-1} g}\right] \tag{4}$$

where $f = \cosh \pi (W/H)_{se}/2$ and $r = 1/\pi$ for $\epsilon_r \geqslant 6$; $r = 8/\pi(\epsilon_r + 2)$ for $\epsilon_r \leqslant 6$.

Although (4) appears quite formidable, it can easily be solved for the variable g by substitution. For any given pair of single strip shape ratios $(W/H)_{so}$ and $(W/H)_{se}$, g is always positive and greater than unity. Fig. 2 is intended as an aid to solve (4). It shows a graph of g as a function of $(W/H)_{so}$ and $(W/H)_{se}$ for a range of practical values. The curves are plotted for $\epsilon_r = 2.2$ and $\epsilon_r \geqslant 6$, but should provide good starting values of g for all ϵ_r.

The value of g which satisfies (4) is used to obtain the final coupler dimensions [10]

$$(W/H) = \frac{1}{\pi}(\cosh^{-1} h - \cosh^{-1} g) \tag{5}$$

$$(S/H) = \frac{2}{\pi}(\cosh^{-1} g) \tag{6}$$

where

$$h = \frac{(g+1)\cdot f}{2} + \frac{(g-1)}{2}.$$

III. RESULTS

The procedure outlined above was verified by comparing results with published data for a few couplers over a range of coupling values and dielectric constants. Couplers with two, four, and six strips were considered. When the number of strips is taken as two, ordinary coupled microstripline parameters accurate to within 6 percent of the Bryant and Wiess data [14] are obtained [12]. Table I summarizes the comparison.

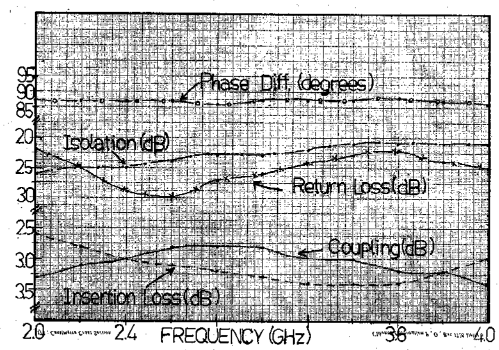

Fig. 3. Performance of a six-strip Lange coupler on 0.020 in RT Duroid 5880 substrate.

The efficacy of this approach was also tested by designing six-strip Lange couplers on RT Duroid 5880 substrate having a dielectric constant of 2.2. The advantage of using six strips for this dielectric lies in the increased spacing and decreased width (both by roughly two and a half times) with respect to a comparable four-strip coupler. This results in a conductor pattern that is much easier to realize with normal printing techniques.

For a coupling of 3 dB and a dielectric thickness of 0.0508 cm, the width and spacing were calculated for a terminating impedance of 50 Ω: $W = 0.0112$ cm; $S = 0.0068$ cm. Undercut in etching was negligible and the dimesions were confirmed by measuring with a microscope. Ultrasonic wire bonding was used to connect alternate strips by single 1-mil aluminum wires.

The performance of this coupler (Fig. 3) shows a maximum amplitude imbalance of 0.7 dB between the two outputs for the octave 2–4 GHz with a midband coupling of 2.8 dB. The response of the coupler is thus overcoupled by 0.2 dB, causing an overlap between coupled and direct outputs which is often desirable [8]. The minimum input return loss is 22 dB and the minimum isolation is 21 dB. Phase difference between outputs deviates from true quadrature by at most 4°. The performance was found to be repeatable.

IV. FINITE CONDUCTOR THICKNESS

The synthesis procedure described above assumed coupled lines of zero conductor thickness. Neglecting finite conductor thickness in design is known to result in couplers with overcoupled responses. To counter this, Presser [5] suggested an empirical technique which increases zero thickness design spacing and reduces zero thickness design width by an equal amount determined from Wheeler's edge correction for single strips of small thicknesses [15].

For the coupler fabricated above, overcoupling caused by assuming $T=0$ was estimated by using Presser's method. Since the nominal T/H ratio of the substrate used is 0.07 (1 oz copper), the equation as given by Presser [5] cannot be applied straightaway; instead, Wheeler's edge correction for single strips of moderately large thicknesses is applicable [13]. (This gives marginally better results.) In analysis form, the correction (ΔW) can be expressed as

$$\Delta W/H=\frac{T/H}{\pi\sqrt{\epsilon}}\cdot\log_e\frac{10.872}{\left[(T/H)^2+\left(\frac{1/\pi}{(W/H)/(T/H)+1.10}\right)^2\right]^{1/2}} \tag{7}$$

where $1/\sqrt{\epsilon}$ is an interpolation factor proposed by Presser [5]. A small error results if ϵ is taken as the effective dielectric constant of a strip of shape ratio W/H.

Equation (7) was used to determine the effective zero thickness width (W') and zero thickness spacing (S') from the actual dimensions of the designed coupler: $W'=0.0153$ cm and $S'=0.0027$ cm. These dimensions indicate a coupling of 2.1 dB and Z_0 of 43 Ω. In other words, a 2.1-dB coupler synthesized for $Z_0=43$ Ω and corrected for a conductor thickness $T/H=0.07$, would have the same dimensions as a 3-dB coupler synthesized for $Z_0=50$ Ω and $T=0$.

A second coupler was fabricated on an identical substrate and a repeatable midband coupling of 2.4 dB was measured. Effective zero thickness dimensions were again determined from the actual dimensions. The coupling if thickness correction is included was estimated to be 1.2 dB and as in the earlier case, this is much higher than the measured value.

Other interpolation factors suggested as $1/\epsilon_r$ by Wheeler [15] and $(1/2+1/2\cdot\epsilon_r)$ by Wheeler [13] were tried with limited success. An interpolation factor $1/2\epsilon_r$ fits experimental results here but it's validity for the general case is doubtful. It is therefore clear that Presser's method which gave excellent results for high dielectric constant substrates and small values of conductor thicknesses cannot be extended to substrates of low dielectric constants and/or large conductor thicknesses. Increasing the zero thickness design gap and reducing zero thickness design width by the same amount is perhaps somewhat arbitrary. The effect of this is obviously not noticeable if ΔW is small (high ϵ_r; small T/H).

V. Conclusions

An approximate synthesis technique for Lange couplers has been shown to be fairly accurate and quite simple to use. The results obtained with the six-strip coupler demonstrate the effectiveness of this approach and are believed to be new for a low dielectric constant laminate. An empirical correction for finite conductor thickness was found to lack general validity.

Appendix

Ou's analysis formulas [9] are

$$Z_0^2=\frac{Z_{0e}\cdot Z_{0o}\cdot(Z_{0o}+Z_{0e})^2}{[Z_{0e}+(k-1)\cdot Z_{0o}]\cdot[Z_{0o}+(k-1)\cdot Z_{0e}]} \tag{8}$$

$$c=\frac{(k-1)\cdot Z_{0e}^2-(k-1)\cdot Z_{0o}^2}{(k-1)\cdot(Z_{0e}^2+Z_{0o}^2)+2\cdot Z_{0e}\cdot Z_{0o}}. \tag{9}$$

Multiplying both sides of (8) by $(1-c/1+c)$ and rearranging

$$Z_{0o}=Z_0\cdot\left(\frac{1-c}{1+c}\right)^{1/2}\cdot\left[1+(k-1)\cdot\frac{Z_{0e}}{Z_{0o}}\right]\cdot\left[1+\frac{Z_{0e}}{Z_{0o}}\right]^{-1}. \tag{10}$$

Equation (2a) can be solved for Z_{0e}/Z_{0o}

$$\frac{Z_{0e}}{Z_{0o}}=\frac{(c+q)}{(k-1)\cdot(1-c)} \tag{11}$$

where

$$q=\left[c^2+(1-c^2)\cdot(k-1)^2\right]^{1/2}.$$

Substituting (4a) in (3a)

$$Z_{0o}=Z_0\cdot\left(\frac{1-c}{1+c}\right)^{1/2}\cdot\frac{(1+q)\cdot(k-1)}{(c+q)+(k-1)\cdot(1-c)}. \tag{12}$$

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